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Statistical analysis of Linear Stability Continuous Time Series Between Two Vector Valued Stochastic Process

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Abstract—The continuous expanded finite Fourier transform of strictly stability (r+s) vector-valued time series are considered, under the assumption that some of the observations are missed . The asymptotic moments are studied. We will our theoretical study to two cases Economy and Electricity Energy.

Index Terms— Finite Fourier transform, Missing values, Data window, Continuous Stability time series.

1 Introduction

We consider the problem the selection of s - vector, μ, and $s\times r$ filter so that $Y(t)\approx\sum a(t-u)\,X(t)$, assuming that there is linear relation between $X(t)$ and $Y(t)$, we study the Asymptotic properties of expanded finite Fourier transform under this relation the expanded finite Fourier transform discussed in D.R. Brillinger, M. Rosenblatt, (1967), D.R. Brillinger (1969), Brillinger (2001), Ghazal and Farag (2001), Ghazal (2002), Teamah (2004), Ghazal, Farag and El-Desokey (2005), M.A. Ghazal, G.S. Mokaddis, A.E.EL-Desokey (2009), G.S. Mokaddis, M.A. Ghazal and A.E. El-Desokey (2010), A. Elhassanein(2011), (2014).

The paper is organized as follows : In Section(1) Introduction, Section (2) we will study the Asym- ptotic properties of the (observed) process, Section (3) will be considered the expanded finite Fourier transform with missed observations, and Section (4) application our theoretical study where we apply this method in the Arab Cement Company of monthly production and quantity of cement sold in the period from January 2010 until December 2015 and in General Electric Company of monthly Sent Energy and export Energy in the period from January 2006 until December 2015.

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2 The Asymptotic Properties the Observed Process

Consider an $(r+s)$ vector-valued stability series

$$B(t)=\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix}, \tag{2.1}$$

$t=0,\pm1,\pm2,.....$ with $X(t)$, r vector-valued and $Y(t)$, s vector-valued a strictly stability $(r+s)$ vector-valued series with components ,

$\begin{bmatrix} X_j(t) \\ Y_i(t) \end{bmatrix}$, $j=1,2,...,r, i=1,2,.....,s$ all of whose moments exist, and we define the means

$$EX(t)=0\,,\ EY(t)=0\ , \tag{2.2}$$

The covariances

$$\begin{aligned} &E\{[X(t+u)-C_x][X(t)-C_x]^T\}=C_{xx}(u)\ , \\ &E\{[X(t+u)-C_x][Y(t)-C_y]^T\}=C_{xy}(u)\ , \\ &E\{[Y(t+u)-C_y][Y(t)-C_y]^T\}=C_{yy}(u)\ , \end{aligned} \tag{2.3}$$

And the second-order spectral densities

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$$f_{xx}(\lambda)=(2\pi)^{-1}\int_{-\infty}^{\infty}C_{xx}(u)\,Exp\ (-i\lambda u)\,du$$

$$f_{xy}(\lambda)=(2\pi)^{-1}\int_{-\infty}^{\infty}C_{xy}(u)\,Exp\ (-i\lambda u)\,du \quad , \qquad for\ \lambda\in R \qquad (2.4)$$

$$f_{yy}(\lambda)=(2\pi)^{-1}\int_{-\infty}^{\infty}C_{yy}(u)\,Exp\ (-i\lambda u)\,du$$

Let $H_a(t), a=1,2,.....,r\ (t\in R)$ be a process independent of $B(t)$ such that every t

$$P[H_a(t)=1]=p_a \ , \qquad P[H_a(t)=0]=q_a \ , \qquad (2.5)$$

Note that $$E\{H_a(t)\}=P \ , \qquad (2.6)$$

The success of recording an observation not depend on the fail of another and so it is independent . We may then define the modified series as

$$W(t)=H(t)B(t) \quad , \qquad (2.7)$$

Where

$$W_a(t)=H_a(t)B_a(t) \quad , \qquad (2.8)$$

And

$$H_a(t)=\begin{cases}1 \ , \text{ if } X_a(t),Y_a(t) \text{ are observed}\\ 0 \ , \ otherwise\end{cases} \quad , \ (2.9)$$

Assumption:

Let $h_a^{(T)}(t)$ be a bounded and bounded variation and vanishes for $0<t<T-1$.That is called data window and satisfies

$$\frac{1}{T}\int_0^T h_a^{\ (T)}\ d_t \underset{T\to\infty}{\longrightarrow}\int_0^1 h_a(u)\,du \ , \qquad a=\overline{1,r} \qquad ,$$

$$G^{(T)}{}_{a_1,\dots,a_k(\lambda)}=\int_0^T\left[\prod_{j=1}^{k}h_{a_j}^{(T)}(t)\ \right]\exp\{-i\lambda t\}\,dt \qquad ,$$

We will now select of an s -vector , $\underline{\mu}$, and an $s\times r$ filter $\{a(u)\}$, so that

$$Y(t)\approx\underline{\mu}+\sum_{u=-\infty}^{\infty}a(t-u)X(u) \qquad (2.10)$$

which is close to $Y(t)$. Suppose we measure closeness by the $S\times S$ Hermitian matrix

$$E\left\{[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)][Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]^T\right\}, \ (2.11)$$

Theorem 2.1 [7]:
Consider an $(r+s)$ vector-valued second-order stability time series of the form (2.1) with mean (2.2) and autocovariance function (2.3), suppose $c_{xx}(u)$, $c_{yy}(u)$ are absolutely summable and suppose $f_{xx}(\lambda)$ given by (2.4), is nonsingular, $\lambda\in R$. Then the, $\underline{\mu}$, and $a(u)$ that minimize (2.11) are given by

$$\underline{\mu}=c_y-\left(\sum_{u=-\infty}^{\infty}a(u)\right)c_x=c_y-A(0)c_x \quad , \qquad (2.12)$$

and

$$a(u)=(2\pi)^{-1}\int_0^{2\pi}A(\alpha)\,Exp\{iu\alpha\}\,d\alpha \quad , \qquad (2.13)$$

where ,

$$A(\lambda)=f_{yx}(\lambda)\,f_{xx}(\lambda)^{-1} \qquad (2.14)$$

The filter $\{a(u)\}$ is absolutely summable. The minimum achieved is

$$\int_0^{2\pi}[f_{yy}(\alpha)-f_{yx}(\alpha)f_{xx}(\alpha)^{-1}f_{xy}(\alpha)]d\alpha. \qquad (2.15)$$

3 The Expanded Finite Fourier Transform With Missed Observations and Its Properties

Theorem 3.1

Let $W_a(t)=H_a(t)B_a(t)$, $a=1,2,......,\min(r,s)$ are missed observations on the stable stochastic processes $X_a(t),Y_a(t)$, $a=1,2,......,\min(r,s)$ and $H_a(t)$ is Bernoulli sequence of random variables *which satisfies equations(2.8)and (2.9) , Then*

$$E\{W_a(t)\}=0 \qquad , \qquad (3.1)$$

$$Cov\{W_{a_1}(t_1),W_{a_2}(t_2)\}=\begin{bmatrix}P_{a_1a_2}c_{xx}(u) & P_{a_1a_2}c_{xx}(u)A(\lambda)^T\\ P_{a_1a_2}A(\lambda)c_{xx}(u) & P_{a_1a_2}A(\lambda)c_{xx}(u)A(\lambda)^T\end{bmatrix} \qquad (3.2)$$

,

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Proof.

Since $H_a(t)$ is independent of $B_a(t)$, then (3.1) is obtained .
Now , turned to (3.2) we have

$$Cov\{W_{a_1}(t_1),W_{a_2}(t_2)\}=Cov\{H_{a_1}(t)B_{a_1}(t),H_{a_2}(t)B_{a_2}(t)\}$$

$$=Cov\left\{\begin{bmatrix}H_{a_1}(t_1)X_{a_1}(t_1)\\ H_{a_1}(t_1)Y_{a_1}(t_1)\end{bmatrix},\begin{bmatrix}H_{a_2}(t_2)X_{a_2}(t_2)\\ H_{a_2}(t_2)Y_{a_2}(t_2)\end{bmatrix}^T\right\}$$

$$=E\begin{Bmatrix}H_{a_1}(t_1)X_{a_1}(t_1)H_{a_2}(t_2)X_{a_2}(t_2) & H_{a_1}(t_1)X_{a_1}(t_1)H_{a_2}(t_2)Y_{a_2}(t_2)\\ H_{a_1}(t_1)Y_{a_1}(t_1)H_{a_2}(t_2)X_{a_2}(t_2) & H_{a_1}(t_1)Y_{a_1}(t_1)H_{a_2}(t_2)Y_{a_2}(t_2)\end{Bmatrix}$$

$$=\begin{Bmatrix}E[H_{a_1}(t_1)H_{a_2}(t_2)]Cov[X_{a_1}(t_1),X_{a_2}(t_2)] & E[H_{a_1}(t_1)H_{a_2}(t_2)]Cov[X_{a_1}(t_1),Y_{a_2}(t_2)]\\ E[H_{a_1}(t_1)H_{a_2}(t_2)]Cov[Y_{a_1}(t_1),X_{a_2}(t_2)] & E[H_{a_1}(t_1)H_{a_2}(t_2)]Cov[Y_{a_1}(t_1),Y_{a_2}(t_2)]\end{Bmatrix}$$

$$=\begin{Bmatrix}p_{a_1a_2}Cov[X_{a_1}(t_1),X_{a_2}(t_2)] & p_{a_1a_2}Cov[X_{a_1}(t_1),\mu+A(\alpha)X_{a_2}(t_2)]\\ p_{a_1a_2}Cov[\mu+A(\alpha)X_{a_1}(t_1),X_{a_2}(t_2)] & p_{a_1a_2}Cov[\mu+A(\alpha)X_{a_1}(t_1),\mu+A(\alpha)X_{a_2}(t_2)]\end{Bmatrix}$$

From equation (2.3) we have,

$$=\begin{bmatrix}P_{a_1a_2}c_{xx}(u) & P_{a_1a_2}c_{xx}(u)A(\lambda)^T\\ P_{a_1a_2}A(\lambda)c_{xx}(u) & P_{a_1a_2}A(\lambda)c_{xx}(u)A(\lambda)^T\end{bmatrix}$$

Then equation (3.2) obtained .

Theorem 3.2

Let $W_a(t)=H_a(t)B_a(t)$ *,* $a=1,2,......,\min(r,s)$ *are missed observations on the stable stochastic processes* $X_a(t),Y_a(t)$ *,* $a=1,2,......,\min(r,s)$ *and* $H_a(t)$ *is Bernoulli sequence of random variables which satisfies equations(2.8)and (2.9) , and* $h_a^{(T)}(t)$ *satisfies Assumption. We define the continuous expanded finite Fourier transform by*

$$d_a^{(T)}(\lambda)=\left[2\pi\int_0^T\left(h_a^{(T)}(t)\right)^2\right]^{-1/2}\times\int_{-\infty}^{\infty}h_a^{(T)}(t)\,W_a(t)\exp\{-i\lambda t\}dt,\ for\ \lambda\in R \quad ,(3.3)$$

For large T , this variate will be distributed approximately as complex Normal Distribution as

$$N^c_{r+s}\left(\underline{0}\ ,\begin{bmatrix}k_1 & k_2\\ k_3 & k_4\end{bmatrix}\right) \qquad (3.4)$$

Where

$$k_1=P_{a_1a_2}\int_R f_{a_1a_2}(v)\phi_{a_1a_2}{}^{(T)}(\lambda_1-v,\lambda_2-v)dv \quad ,$$

$$k_2=P_{a_1a_2}\int_R f_{a_1a_2}(v)A(\lambda)^T\phi_{a_1a_2}{}^{(T)}(\lambda_1-v,\lambda_2-v)dv \quad ,$$

$$k_3=P_{a_1a_2}\int_R A(\lambda)f_{a_1a_2}(v)\phi_{a_1a_2}{}^{(T)}(\lambda_1-v,\lambda_2-v)dv \quad ,$$

$$k_4=P_{a_1a_2}\int_R A(\lambda)f_{a_1a_2}(v)A(\lambda)^T\phi_{a_1a_2}{}^{(T)}(\lambda_1-v,\lambda_2-v)dv \ ,$$

Proof.

From equations (3.1) and (3.3), then

$$E\{d_a(t)\}=0 \quad ,$$

and

$$Cov\{d_{a_1}^{(T)}(\lambda_1),d_{a_2}^{(T)}(\lambda_2)\}=E\{d_{a_1}^{(T)}(\lambda_1)\overline{d_{a_2}^{(T)}(\lambda_2)}\}$$

$$=(2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(h_{a_1}^{(T)}(t_1)\right)^2\left(h_{a_2}^{(T)}(t_2)\right)^2dt_1dt_2\right]^{-1/2}\times$$

$$\times\int_{t_1=0}^{T}\int_{t_2=0}^{T}h_{a_1}(t_1)h_{a_2}(t_2)Cov(W_{a_1}(t_1),W_{a_2}(t_2))\exp(-i(t_1\lambda_1-t_2\lambda_2)dt_1dt_2$$

From (3.2) then we get

$$Cov\{d_{a_1}^{(T)}(\lambda_1),d_{a_2}^{(T)}(\lambda_2)\}=(2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(h_{a_1}^{(T)}(t_1)\right)^2\left(h_{a_2}^{(T)}(t_2)\right)^2\right]^{-1/2}\times$$

$$\times\int_{t_1=0}^{T}\int_{t_2=0}^{T}h_{a_1}(t_1)h_{a_2}(t_2)\exp(-i(t_1\lambda_1-t_2\lambda_2)dt_1dt_2\times$$

$$\times\begin{bmatrix}P_{a_1a_2}c_{xx}(t_1-t_2) & P_{a_1a_2}c_{xx}(t_1-t_2)A(\lambda)^T\\ P_{a_1a_2}A(\lambda)c_{xx}(t_1-t_2) & P_{a_1a_2}A(\lambda)c_{xx}(t_1-t_2)A(\lambda)^T\end{bmatrix}$$

Setting $t_1-t_2=u\,,t_2=t$ we get

$$Cov\{d_{a_1}^{(T)}(\lambda_1),d_{a_2}^{(T)}(\lambda_2)\}=(2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(h_{a_1}^{(T)}(t_1)\right)^2\left(h_{a_2}^{(T)}(t_2)\right)^2dt_1dt_2\right]^{-1/2}\times$$

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$$\times \int_{-T}^{T}\int_{0}^{T} h_{a_1}(t+u)h_{a_2}(t)\exp(-it(\lambda_1-\lambda_2)dtdu \times$$

$$\begin{bmatrix} P_{a_1a_2}c_{xx}(u) & P_{a_1a_2}c_{xx}(u)A(\lambda)^T \\ P_{a_1a_2}A(\lambda)c_{xx}(u) & P_{a_1a_2}A(\lambda)c_{xx}(u)A(\lambda)^T \end{bmatrix} = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} \qquad (3.6)$$

Now :

$$C_{xx}(u) = E\{X(t+u)X(t)\} = \int_{-\infty}^{\infty} f_{xx}(\lambda)\exp(i\lambda u) \qquad (3.7)$$

By substituting a bout formula (3.7) in (3.5) we get

$$k_1 = p_{a_1a_2}\int_{-\infty}^{\infty} f_{a_1a_2}(v)\exp(iv(t_1-t_2)(2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(h_{a_1}^{(T)}(t_1)\right)^2\left(h_{a_2}^{(T)}(t_2)\right)^2 dt_1dt_2\right]^{-1/2} \times$$

$$\times\left\{\int_{t_1=0}^{T}\int_{t_2=0}^{T} h_{a_1}(t_1)h_{a_2}(t_2)\exp(-i[(t_1\lambda_1 - t_2\lambda_2)]\,dt_1dt_2\right\}dv$$

$$= p_{a_1a_2}\int_{-\infty}^{\infty} f_{a_1a_2}(v)(2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(h_{a_1}^{(T)}(t_1)\right)^2\left(h_{a_2}^{(T)}(t_2)\right)^2 dt_1dt_2\right]^{-1/2} \times$$

$$\times\left\{\int_{t_1=0}^{T}\int_{t_2=0}^{T} h_{a_1}(t_1)h_{a_2}(t_2)\exp(-i[(\lambda_1-v)t_1-(\lambda_2-v)t_2]\,dt_1dt_2\right\}dv$$

$$k_1 = p_{a_1a_2}\int_{-\infty}^{\infty} f_{a_1a_2}(v)\times\phi_{a_1a_2}(\lambda_1-v,\lambda_2-v)dv$$

where

$$\phi_{a_1a_2}(\lambda_1-v,\lambda_2-v) = (2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(h_{a_1}^{(T)}(t_1)\right)^2\left(h_{a_2}^{(T)}(t_2)\right)^2 dt_1dt_2\right]^{-1/2} \times$$

$$\times\left\{\int_{-T}^{T}\int_{0}^{T} h_{a_1}{}^{(T)}(t+u)h_{a_2}{}^{(T)}(t)\exp(-i[(\lambda_1-v)t-(\lambda_2-v)t]\,dtdtu\right\}$$

Similarly

$$k_2 = p_{a_1a_2}\int_{-\infty}^{\infty} f_{a_1a_2}(v)A(\lambda)^T\times\phi_{a_1a_2}(\lambda_1-v,\lambda_2-v)dv \quad ,$$

$$k_3 = p_{a_1a_2}\int_{-\infty}^{\infty} A(\lambda)f_{a_1a_2}(v)\times\phi_{a_1a_2}(\lambda_1-v,\lambda_2-v)dv \ ,$$

$$k_4 = p_{a_1a_2}\int_{-\infty}^{\infty} A(\lambda)f_{a_1a_2}(v)A(\lambda)^T\times\phi_{a_1a_2}(\lambda_1-v,\lambda_2-v)dv \quad ,$$

Then equation (3.4) is obtained .

Corollary 3.1

Let $d_a^{(T)}(\lambda), a=1,\dots\dots, \min(s,r)$ *be defined as (3.3) , then the dispersion of* $d_a^{(T)}(\lambda)$ *satisfies the following property :*

$$Dd_a^{(T)}(\lambda) = P_{aa} \times$$

$$\times\begin{bmatrix} \int_{-\infty}^{\infty} f_{aa}(\lambda-\gamma)\times\phi_{aa}(\gamma)\,d\gamma & \int_{-\infty}^{\infty} f_{aa}(\lambda-\gamma)\,A(\lambda)^T\times\phi_{aa}(\gamma)\,d\gamma \\ \int_{-\infty}^{\infty} A(\lambda)\,f_{aa}(\lambda-\gamma)\times\phi_{aa}(\gamma)\,d\gamma & \int_{-\infty}^{\infty} A(\lambda)\,f_{aa}(\lambda-\gamma)\,A(\lambda)^T\times\phi_{aa}(\gamma)\,d\gamma \end{bmatrix} \quad (3.8)$$

Proof.

Form equation (3.4) we have

$$Dd_a^{(T)}(\lambda) =$$

$$p_{a_1a_2}\times\begin{bmatrix} \int_{-\infty}^{\infty} f_{a_1a_2}(v)\times\phi_{a_1a_2}(\lambda_1-v,\lambda_2-v)dv & \int_{-\infty}^{\infty} f_{a_1a_2}(v)A(\lambda)^T\times\phi_{a_1a_2}(\lambda_1-v,\lambda_2-v)dv \\ \int_{-\infty}^{\infty} A(\lambda)f_{a_1a_2}(v)\times\phi_{a_1a_2}(\lambda_1-v,\lambda_2-v)dv & \int_{-\infty}^{\infty} A(\lambda)f_{a_1a_2}(v)A(\lambda)^T\times\phi_{a_1a_2}(\lambda_1-v,\lambda_2-v)dv \end{bmatrix}$$

When $\lambda_1=\lambda_2=\lambda, \lambda\in R$ and $a_1=a_2=a, a=1,2,\dots\dots, \min(r,s)$, by putting $\lambda - v = \gamma$, then formula (3.8) is obtained .

Lemma 3.1

Let $h_a^{(T)}(t), t\in R, a=1,\dots\dots, \min(r,s)$ *is bounded by a constant* L *and satisfying*

$$\left|h_a^{(T)}(t+u) - h_a(t)\right| \le C|u| \quad ,$$

Then

$$\left|\int_0^T h_{a_1}{}^{(T)}(t)h_{a_2}{}^{(T)}(t)\exp(-i\lambda t)\,dt\right| \le \frac{1}{|\lambda/2|} + LC \ , \quad (3.9)$$

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For some constants L, C *and* $\lambda,\lambda\in R,\lambda\neq 0,\ a_1,a_2 = 1,\ldots,\min(r,s)$.

Lemma 3.2

For $\lambda_1-\lambda_2\neq 0,\lambda_1,\lambda_2\in R$ *and* $h_a^{(T)}(t),t\in R,a=1,\ldots\ldots,\min(r,s)$ *is bounded by constant* L *and satisfying Lipschitz condition (3.9) then*

$$\left|Cov\left\{d_{a_1}^{(T)}(\lambda_1),d_{a_2}^{(T)}(\lambda_2)\right\}\right|\leq \frac{LC}{2\pi\sqrt{\int_0^T\int_0^T(h_{a_1}^{(T)}(t_1))^2(h_{a_2}^{(T)}(t_2))^2\,dt_1dt_2}}\times$$

$$\times\left\{\frac{1}{LC\left|(\lambda_1-\lambda_2)/2\right|}\int_{-T}^{T}\left|C_{a_1a_2}(u)\right|du+\int_{-T}^{T}\left|C_{a_1a_2}(u)\right|\left[\left|u\right|+1\right]du\right\} \quad (3.10)$$

,

for $a_1,a_2 = 1,\ldots\ldots,\min(r,s)$

Theorem 3.3

For $\lambda_1-\lambda_2\neq 0,\lambda_1,\lambda_2\in R$ and $h_a^{(T)}(t),t\in R,a=1,\ldots\ldots,\min(r,s)$ is bounded and

$$\int_{-\infty}^{\infty}\left[\left|u\right|+1\right]C_{a_1a_2}(u)\left|du<\infty\right. ,$$

then

$$\lim_{T\to\infty} Cov\left\{d_{a_1}^{(T)}(\lambda_1),d_{a_2}^{(T)}(\lambda_2)\right\}=0 \quad \text{for all}$$

$a_1,a_2=1,\ldots\ldots,\min(r,s)$

Proof.

The proof comes directly from Lemma (3.2) and Assumption.

Theorem 3.4

For any $\lambda\in R$, *the function* $\phi_{aa}^{(T)}(\lambda),a=1,\ldots\ldots,\min(r,s)$ *is the Kernel that satisfy the following properties :*

(1) $$\int_{-\infty}^{\infty}\phi_{aa}^{(T)}(\lambda)d\lambda=1,\ a=1,\ldots\ldots,\min(r,s) \quad , \quad (3.11)$$

(2) $$\lim_{T\to\infty}\int_{-\infty}^{-\delta}\phi_{aa}^{(T)}(\lambda)d\lambda=\lim_{T\to\infty}\int_{\delta}^{\infty}\phi_{aa}^{(T)}(\lambda)d\lambda=0,\quad (3.12)$$

for $\delta>0,a=1,\ldots\ldots,\min(r,s)\ \lambda\in R$,

(3) $$\lim_{T\to\infty}\int_{-\delta}^{\delta}\phi_{aa}^{(T)}(\lambda)d\lambda=1 \textit{ for all}$$

$$\delta>0,a=1,\ldots\ldots,\min(r,s)\ \lambda\in R \ . \quad (3.13)$$

Theorem 3.5

If the spectral density function $f_{aa}(x),a=1,\ldots.,\min(r,s),x\in R$ *is bounded and continuous at a point* $x=\lambda,\lambda\in R$ *and the function* $\phi_{aa}^{(T)}(x)$, *satisfies the properties of theorem* $a=1,\ldots.,\min(r,s),x\in R$ *(3.4) then*

$$\lim_{T\to\infty} Dd_a^{(T)}(\lambda)=\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T\end{bmatrix},\ a=1,\ldots\ldots,\min(r,s) \quad (3.14)$$

Proof.

To prove formula (3.14), we must prove that

$$\lim_{T\to\infty}\left|Dh_a^{(T)}(\lambda)-p_{aa}\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T\end{bmatrix}\right|=0,$$

Now , from corollary (3.1) we have,

$$\left|Dh_a^{(T)}(\lambda)-p_{aa}\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T\end{bmatrix}\right|\leq$$

$$\leq p_{aa}\int_{-\infty}^{\infty}\left\|\begin{bmatrix} f_{aa}(\lambda-\gamma) & f_{aa}(\lambda-\gamma)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T\end{bmatrix}-\right.$$

$$\left.\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T\end{bmatrix}\right\|\Omega_{aa}^{(T)}(\gamma)d\gamma\leq$$

$$\leq p_{aa}\int_{-\infty}^{-\delta}\left\|\begin{bmatrix} f_{aa}(\lambda-\gamma) & f_{aa}(\lambda-\gamma)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T\end{bmatrix}-\right.$$

$$\left.-\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T\end{bmatrix}\right\|\Omega_{aa}^{(T)}(\gamma)d\gamma+$$

$$+\ p_{aa}\int_{-\delta}^{\delta}\left\|\begin{bmatrix} f_{aa}(\lambda-\gamma) & f_{aa}(\lambda-\gamma)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T\end{bmatrix}-\right.$$

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$$-\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\Bigg|\Omega_{aa}^{(T)}(\gamma)d\gamma +$$

$$+ p_{aa}\int_{\delta}^{\infty}\Bigg|\begin{bmatrix} f_{aa}(\lambda-\gamma) & f_{aa}(\lambda-\gamma)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T \end{bmatrix} -$$

$$-\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\Bigg|\Omega_{aa}^{(T)}(\gamma)d\gamma$$

$$= A_1 + A_2 + A_3 .$$

Since $f_{a_1a_2}(\gamma)$ is continuous at a point $\gamma=\lambda, a_1, a_2 = 1,\dots,\min(r,s)$, $\lambda\in R$, then we get

$$A_2 = p_{aa}\int_{-\delta}^{\delta}\Bigg|\begin{bmatrix} f_{aa}(\lambda-\gamma) & f_{aa}(\lambda-\gamma)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T \end{bmatrix} -$$

$$-\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\Bigg|\Omega_{aa}^{(T)}(\gamma)d\gamma$$

$$= p_{aa}\int_{-\delta}^{\delta}\left\|\begin{matrix} f_{aa}(\lambda-\gamma)-f_{aa}(\lambda) & f_{aa}(\lambda-\gamma)A(\alpha)^T - f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma)-A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T - A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{matrix}\right\| \times$$

$$\leq \varepsilon\int_{-\delta}^{\delta}\Omega_{aa}^{(T)}(\gamma)d\gamma \leq \varepsilon\int_{-\infty}^{\infty}\Omega_{aa}^{(T)}(\gamma)d\gamma \times \Omega_{aa}^{(T)}(\gamma)d\gamma$$

,

Hence, $A_2 \leq \varepsilon$. Now A_2 is very small according to any ε is very small, consequently $A_2 = 0$. Suppose that $f_{aa}(\lambda)$ $a=1,\dots,\min(r,s), \lambda\in R$ is bounded by a constant M, then

$$A_1 \leq 2M\int_{-\infty}^{-\delta}\Omega_{aa}^{(T)}(\gamma)d\gamma \xrightarrow[T\to\infty]{} 0,$$

according to property (3.12). similarly $A_3 \xrightarrow[T\to\infty]{} 0$, therefore,

$$\left|Dh_a^{(T)}(\lambda) - p_{aa}\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\right| \xrightarrow[T\to\infty]{} 0.$$

which completes the proof of the theorem.

4. Application of Our Theoretical Study

We will apply our theoretical study to a practical cases in Economy and Electricity Energy as following:

4.1. Studying the Production and Cement Sold .

The data available in this research represents the average of the monthly production of Cement producer Arabian cement company and the Cement sold for the period from January 2010 to December 2015.

4.1.1. Studying the Production .

In this study we will comparison between our results, model of strictly stability time series with some missing observations and the classical results, where all observations are available.

Let $\varphi_a(t) = H_a(t)X_a(t)$, $a=1, 2, \dots, r$, where $X_a(t), (t=0, \pm1, \dots)$ be a strictly stability r-vector valued time series and $H_a(t)$ is Bernoulli sequence of random variable which is stochastically independent of $X_a(t)$, we suppose know that the data $X_a(t), (t=(1,2,\dots,T]$ which is the average of the monthly production, where all observations are available of the series is available with some missing observations. $H=1$, $\varphi_a(t) = X_a(t)$, which is the classical case and suppose that there is some missing observations in randomly way, i.e., $H=0$, table (4.1.1) shows comparison these results with and without missed observations.

TABLE4.1.1

COMPARISON OF THE RESULTS WITH AND WITHOUT MISSED OBSERVATIONS OF THE PRODUCTION .

without missed observations	with missed observations

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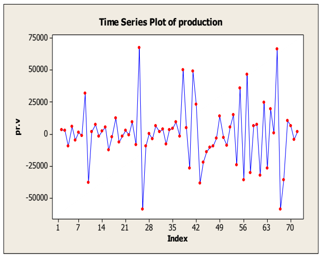
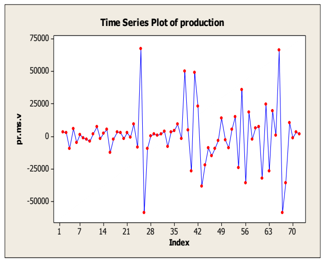
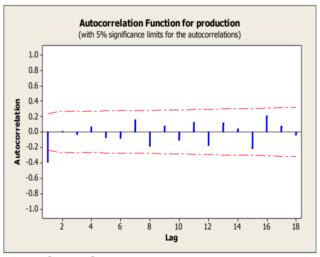


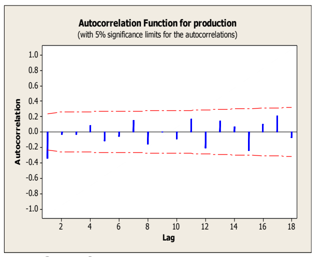
The average monthly production



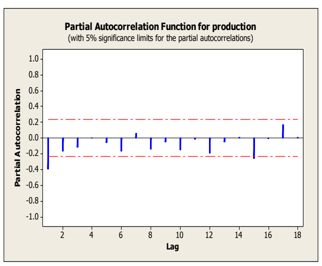
The average monthly production



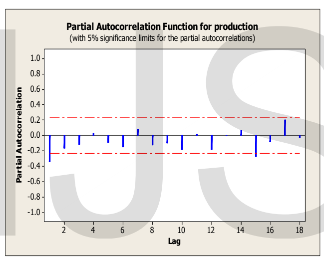
ACF of the average monthly production



ACF of the average monthly production



PACF of the average monthly production



PACF of the average monthly production

ARIMA Model: Production without missed observations

$ARIMA(1,1,1)$

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR 1	0.5007	0.111	4.48	000
MA 1	0.9739	0.0464	20.98	.000

Differencing: 1 regular difference
Number of observations: Original series 72, after differencing 71
Residuals: SS = 30175541940 (back forecasts excluded)
MS = 44375970 DF = 68

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	11.5	21.7	37.7	49.8
DF	9	21	33	45
P-Value	0.241	0.415	0.263	.289

ARIMA Model: Production with missed observations

$ARIMA(1,1,1)$

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR1	0.5595	0.1074	5.21	.000
MA 1	0.9748	0.045	21.35	0.000

Differencing: 1 regular difference
Number of observations: Original series 72, after differencing 71
Residuals: SS = 26976104349 (back forecasts excluded)
MS = 396707417 DF = 68

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	11.4	23.6	40.6	52.5
DF	9	21	33	45
P-Value	0.250	0.315	0.169	0.206

4.1.2. Studying the Cement Sold

In this study we will comparison between our results, model of strictly stability time series with some missing observations and the classical results, where all observations are available.

Let $\psi_a(t) = H_a(t)Y_a(t)$, $a = 1,2,......,s$, where $Y_a(t),(t = 0,\pm 1,.....)$ be a strictly stability s-vector valued time series and $H_a(t)$ is Bernoulli sequence of random variable which is stochastically independent of $Y_a(t)$, we suppose know that the data $Y_a(t),(t = (1,2,.....,T]$ which is the average of the monthly Cement sold, where all observations are available of the series is available with some missing observations. $H = 1$, $\Psi_a(t) = Y_a(t)$, which is the classical case and suppose that there is some missing observations in randomly way, i.e., $H = 0$, table (4.1.2) shows comparison these results with and without missed observations

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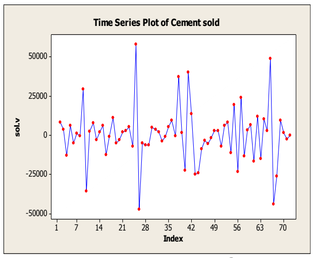
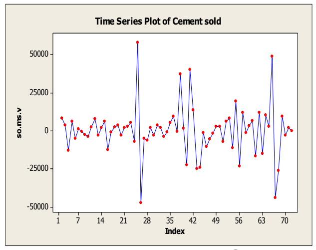
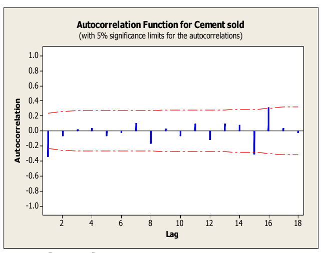
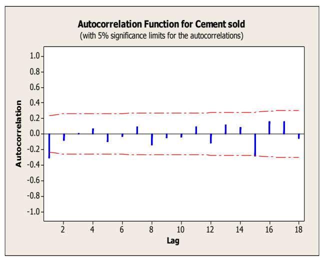
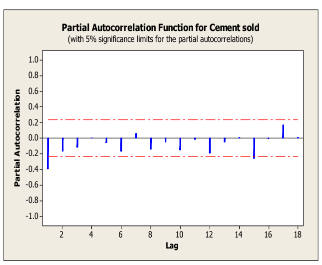
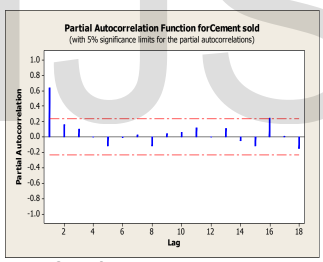
TABLE4.1.2

COMPARISON OF THE RESULTS WITH AND WITHOUT MISSED OBSERVATIONS OF THE CEMENT SOLD

without missed observations	with missed observations
 The average monthly Cement sold	 The average monthly Cement sold
 ACF of the average monthly Cement sold	 ACF of the average monthly Cement sold
 PACF of the average monthly Cement sold	 PACF of the average monthly Cement sold

<table>
<tr>
<td>ARIMA Model: The Cement sold without missed observations
ARIMA(1,1,1)
Final Estimates of Parameters
Typ Coef SE Coef T P
AR 1 0.5058 0.1108 4.57 0.000
AM 1 0.9772 0.0452 21.60 0.000

Differencing: 1 regular difference
Number of observations: Original series 72, after differencing 71
Residuals: SS = 16812512213 (back forecasts excluded)
MS = 247242827 DF = 68

Modified Box-Pierce (Ljung-Box) Chi-Square statistic
Lag 12 24 36 48
Chi-Squar 7.6 23 37.3 50.5
DF 9 21 33 45
P-Value 0.575 0.346 0.279 0.266</td>
<td>ARIMA Model: The Cement sold without missed observations
ARIMA(1,1,1)
Final Estimates of Parameters
Typ Coef SE Coef T P
AR 1 0.5606 0.1070 5.24 0.000
AM 1 0.977 0.0499 21.75 0.000

Differencing: 1 regular difference
Number of observations: Original series 72, after differencing 71
Residuals: SS = 15118432102 (back forecasts excluded)
MS = 222329884 DF = 68

Modified Box-Pierce (Ljung-Box) Chi-Square statistic
Lag 12 24 36 48
Chi-Square 7.5 19.9 35.6 47.7
DF 9 21 33 45
P-Value 0.614 0.526 0.348 0.362</td>
</tr>
</table>

4.1.3. Studying the Regression Between Production and Cement Sold

In this study we will comparison between our results, regression model between Monthly average of Production and average Monthly Cement sold with some missing observations and the classical results, where all observations are available, to compare two cases shown table (4.1.3)

TABLE 4.1.3

THE COMPARISON OF THE RESULTS WITH AND WITHOUT MISSED OBSERVATIONS OF THE REGRESSION ANALYSIS

Without missed observations	With missed observations
The regression equation is	The regression equation is
Cement sold = 3363 + 0.737 Production	Cement sold = 3223 + 0.744 Production
Predictor Coef SE Coef T P	Predictor Coef SE Coef T P
Constant 3362.8 611.2 5.50 0.000	Constant 3323.1 520.7 6.19 0.000
Production 0.73695 0.01348 54.65 0.000	Production 0.74408 0.01172 63.5.84 0.000
S = 2931.16 R-Sq = 97.7% R-Sq(adj) = 97.7%	S = 2931.16 R-Sq = 98.3% R-Sq(adj) = 98.3.6%
Analysis of Variance	Analysis of Variance
Source DF SS MS F P	Source DF SS MS F P
Regression 1 256616 256616 2986.8 0.000	Regressio 1 257679 257679 4032.55 0.000
Residual Error 70 601417 85916	Residual Error 70 447299374 6389991
Total 71 26263033	Total 71 26215254504
Durbin-watson statistic =1.77188	Durbin-watson statistic =1.76825

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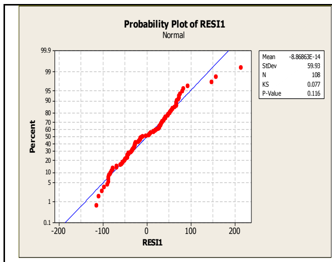
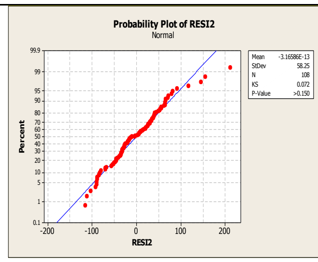
 Normal-plot of standardized Residuals	 Normal-plot of standardized Residuals

4.1.4 Materials and Methods

We used SPSS and MINITAB, the software programming to solve our numerical example .

4.1.5 Results and Discussion

(1) The study of the time series with missed observations had the same results of the study of the classical time series .

(2) The study regression model between classical time series X(t) ,Y(t) had the same results as case of missed observations where the two models achieved the theoretical, Mathematical, and the least squares conditions .

4.2 Studying the Sent Energy and the Export Energy.

The data available in this research represents the average of the monthly sent Energy of General Electric Company and the export Energy for the period from January 2006 to December 2015.

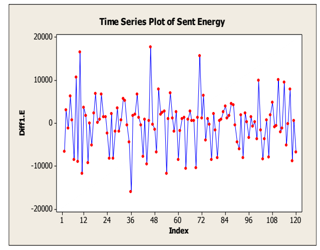
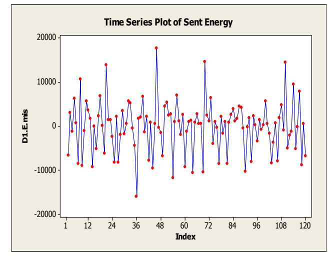
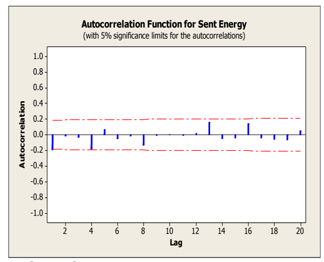
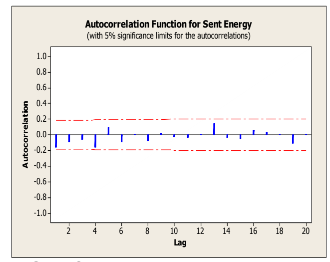
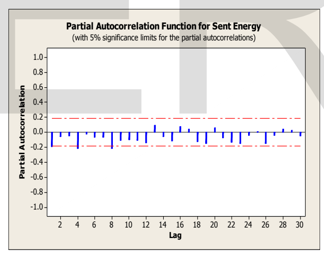
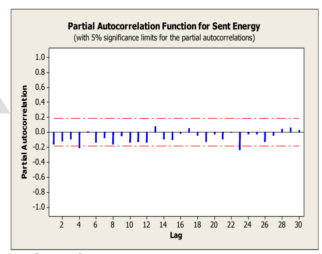
4.2.1 Studying the Sent Energy .

In this study we will comparison between our results, model of strictly stability time series with some missing observations and the classical results, where all observations are available.

Let $\varphi_a(t)=H_a(t)X_a(t), a=1,2,......,r$, where $X_a(t),(t=0,\pm1,.....)$ be a strictly stability r-vector valued time series and $H_a(t)$ is Bernoulli sequence of random variable which is stochastically independent of $X_a(t)$, we suppose know that the data $X_a(t),(t=(1,2,.....,T]$ which is the average of the monthly sent Energy, where all observations are available of the series is available with some missing observations. $H=1$, $\phi_a(t)=X_a(t)$, which is the classical case and suppose that there is some missing observations in randomly way, i.e., $H=0$, table (4.2.1) shows comparison these results with and without missed observations.

TABLE 4.2.1

COMPARISON OF THE RESULTS WITH AND WITHOUT MISSED OBSERVATIONS OF THE ENERGY SENT.

without missed bservations	with missed observations
 The average monthly sent Energy	 The average monthly sent Energy
 ACF of the average monthly sent Energy	 ACF of the average monthly sent Energy
 PACF of the average monthly sent Energy	 PACF of the average monthly sent Energy

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ARIMA Model: Sent Energy without missed observations	**ARIMA Model:** Sent Energy with missed observations
ARIMA(1,1,1)	*ARIMA*(1,1,1)
Final Estimates of Parameters	Final Estimates of Parameters
Type Coef SE Coef T P	Type Coef E Coef T P
AR 1 0.6224 0.0751 8.29 0.000	AR1 0.559 0.0758 7.91 0.000
MA 1 0.9781 0.0200 48.84 0.000	MA1 0.9804 0.0132 74.25 0.000
Differencing: 1 regular difference	Differencing: 1 regular difference
Number of observations: Original series 120, after differencing 119	Number of observations: Original series 120, after differencing 119
Residuals: SS = 3346639081 (back forecasts excluded)	Residuals: SS = 3211731948 (back forecasts excluded)
MS = 28850337 DF = 116	MS = 27687344 DF = 116
Modified Box-Pierce (Ljung-Box) Chi-Square statistic	Modified Box-Pierce (Ljung-Box) Chi-Square statistic
Lag 12 24 36 48	Lag 12 24 36 48
Chi-Square 8.8 19.0 30.3 45.7	Chi-Square 6.1 15.5 27.5 36.4
DF 9 21 33 45	DF 9 21 33 45
P-Value 0.460 0.585 0.602 .443	P-Value 0.731 0.796 0.738 0.815

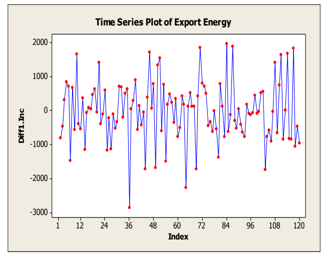
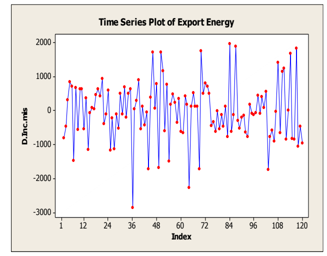
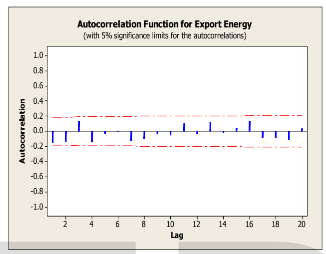
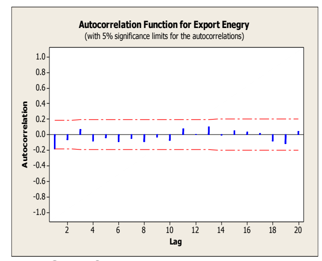
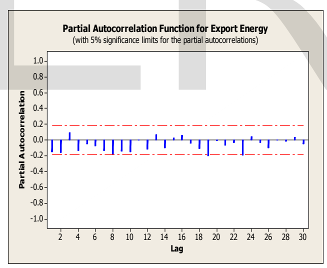
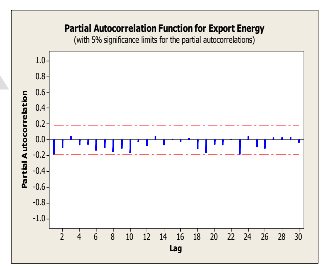
4.2.2 Studying the Export Energy.

In this study we will comparison between our results, model of strictly stability time series with some missing observations and the classical results, where all observations are available.

Let $\Psi_a(t)=H_a(t)Y_a(t)$, $a=1,2,......,s$, where $Y_a(t),(t=0,\pm1,.....)$ be a strictly stability s-vector valued time series and $H_a(t)$ is Bernoulli sequence of random variable which is stochastically independent of $Y_a(t)$, we suppose know that the data $Y_a(t),(t=(1,2,.....,T]$ which is the average of the monthly export energy, where all observations are available of the series is available with some missing observations. $H=1$, $\Psi_a(t)=Y_a(t)$, which is the classical case and suppose that there is some missing observations in randomly way, i.e., $H=0$, table (4.2.2) shows comparison these results with and without missed observations.

TABLE 4.2.2

COMPARISON OF THE RESULTS WITH AND WITHOUT MISSED OBSERVATIONS OF THE EXPORT ENERGY.

without missed observations	with missed observations
 The average monthly export Energy	 The average monthly export Energy
 ACF of the average monthly export Energy	 ACF of the average monthly export Energy
 PACF of the average monthly export Energy	 PACF of the average monthly export Energy

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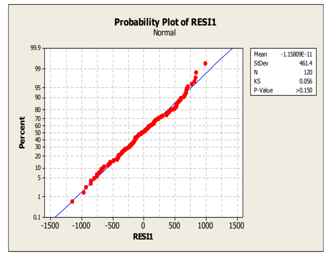
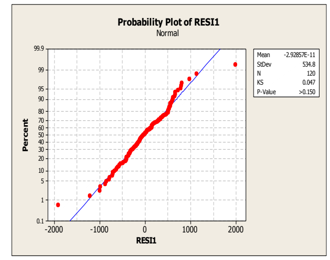
ARIMA Model: The export Energy without missed observations $ARIMA(1,1,1)$	**ARIMA Model:** The export Energy without missed observations $ARIMA(1,1,1)$
Final Estimates of Parameters Type Coef SE Coef T P AR1 0.6766 0.069 99.69 0.000 AM 1 0.9802 0.0143 68.55 0.000	Final Estimates of Parameters Type Coef SE Coef T P AR1 0.6836 0.0706 9.68 0.000 AM 1 0.9784 0.0201 48.57 0.000
Differencing: 1 regular difference Number of observations: Original series 120, after differencing 119 Residuals: SS = 80427160 (back forecasts excluded) MS = 693338 DF = 116	Differencing: 1 regular difference Number of observations: Original series 120, after differencing 119 Residuals: SS = 75289744 (back forecasts excluded) MS = 649050 DF = 116
Modified Box-Pierce (Ljung-Box) Chi-Square statistic Lag 12 24 36 48 Chi-Square 12.2 26.0 36.3 50.7 DF 9 21 33 45 P-Value 0.202 0.205 0.317 0.258	Modified Box-Pierce (Ljung-Box) Chi-Square statistic Lag 12 24 36 48 Chi-Square 8.2 20.0 27.9 45 DF 9 21 33 45 P-Value 0.515 0.522 0.720 0.622



Normal-plot of standardized Residuals



Normal-plot of standardized Residuals

4.2.3 Studying the Regression Between Sent and Export Energy

In this study we will comparison between our results, regression model between Monthly average of sent Energy and average Monthly export Energy with some missing observations and the classical results, where all observations are available, to compare two cases shown table (4.2.3)

TABLE 4.2.3

COMPARISON OF THE RESULTS WITH AND WITHOUT MISSED OBSERVATIONS OF THE REGRESSION ANALYSIS

Without missed observations	Without missed observations
The regression equation is	The regression equation is
Export Energy = 3371 + 0.164 Sent Energy	Export Energy = 2524 + 0.167 Sent Energy
Predictor Coef SE Coef T P	Predictor Coef SE Coef T P
Constant 3371 1352 2.49 0.014	Constant 2524 1685 1.50 0.037
Sent Energy 0.163568 0.005399 30.29 0.000	Sent Energy 0.167028 0.006724 24.84 0.000
S = 463.317 R-Sq = 88.6% R-Sq(adj) = 88.5%	S = 537.098 R-Sq = 83.9% R-Sq(adj) = 83.8%
Analysis of Variance	Analysis of Variance
Source DF SS MS F P	Source DF SS MS F P
Regression 1 197005063 1970050639 17.74 .000	Regression 1 178004361 178004361 617.05 0.000
Residual Error 118 25330185 214663	Residual Error 118 34040021 288475
Total 119 222335249	Total 119 212044382
Durbin-watson statistic = 1.79867	Durbin-watson statistic = 1.66860

4.2.4. Materials and Methods:

We used SPSS and MINITAB, the software programming to solve our numerical example .

4.2.5. Results and Discussion:

(1) The study of the time series with missed observations had the same results of the study of the classical time series .

(2) The study regression model between classical time series X(t) ,Y(t) had the same results as case of missed observations where the two models achieved the theoretical, Mathematical, and the least squares conditions.

REFERENCES

[1] D.R. Brillinger, M.Rosenblatt, Asymptotic theory of estimates of k-th order spectra, in: B. Harris (Ed.), Advanced Seminar on Spectral Analysis of time series, Wiley, New York, 1967, pp.153-188 .

[2] D.R. Brillinger, "Asymptotic properties of spectral estimates of second order", Biometrika56(2), (1969)375-390.

[3] R. Dahlhaus, "On a spectral density estimate obtained by averaging periodograms", J. Appl. Probab. 22(1985) 592-610.

[4] M.A. Ghazal, E.A. Farag, "Estimating the spectral density, autocovariance function and spectral measure of continuous-time stationary processes, in: Annual Conference", ISSR, Cairo, Vol.33, no. part, 1998, pp.1-20.

[5] M.A. Ghazal, "On a spectral density estimate on non-crossed intervals observation", Int. J. Appl. Math.1 (8) (1999) 875-882.

[6] M.A. Ghazal, E.A. Farag, "Some Properties of The Expanded Finite Fourier Transform, Statistica", anno LX, no.3, 2000.

[7] D.R. Brillinger , Time Series Data Analysis and Theory (2001).

International Journal of Scientific & Engineering Research, Volume 8, Issue 4, April-2017
ISSN 2229-5518

[8] A.A.M. Teamah , H.S. Bakouch, " Multitaper Multivriate Spectral Estimators Time Series with Distorted Observations", International Journal of Pure and Applied Mathematics,Vol,No.1(2004),45-57.

[9] Chris Chatfiold, The Analysis of Time series An Introduction .

[10] M. A. Ghazal, E. A. Farag, A.E EL-Desokey," Some properties of the Discrete Expanded finite Fourier transform with missed Observations", Vol.40, No.3, 30 September (2005), pp887-902.

[11] M.A. Ghazal and A. Elhassanein, "Periodogram Analysis with Missing Observations", J.Appl. Math. Computing Vol.22 (2006), No.1-2,pp209-222.

[12] M.A. Ghazal, G.S. Mokaddis, A.E. El-Desokey, "Spectral analysis of strictly stationary continuous time series", Journal of Mathematical Sciences, Vol.3, No.1 (2009).

[13] G.S. Mokaddis, M.A. Ghazal and A.E. El-Desokey, " Asymptotic Properties of Spectral Estimates of Second-Order with Missed Observations", Journal of Mathematics and statistics 6(1):10-16,(2010).

[14] A. ELhassanein, "Nonparametric Spectral Analysis On Disjoint Segments Of Observations", Journal of the Applied Mathematics, Statistics and Informatics(JAMSI),7 (2011),No.1.

[15] A. ELhassanein, "On the Theory of continuous Time series", IndianJ.Pure.Math,45(3):297-310,June(2014). .